Translating Ontologies from Predicate-based to Frame-based Languages

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Outline

Description Logic Programs (DLP)

- Intersection of $\mathcal{SHOIN}$ (OWL DL) and Logic Programming
- Essentially, the Horn subset of $\mathcal{SHOIN}$: $\mathcal{DHL}$ (Description Horn Logic)
- $\mathcal{DHL}$ descriptions:

$$
\begin{array}{lcl}
C, D & \longrightarrow & A \mid C \sqcap D \mid \exists R.\{o\} \\
C_L, D_L & \longrightarrow & C \mid C_L \sqcup D_L \mid \exists R.C_L \mid \geqslant 1 R_L \mid \\
 & & \{o_1, \ldots, o_n\} \\
C_R, D_R & \longrightarrow & C \mid \forall R.C_R
\end{array}
$$

- $\mathcal{DHL}$ axioms:
  $C_L \sqsubseteq D_R \mid C \equiv D \mid R \sqsubseteq S \mid R \equiv S \mid R \equiv S^- \mid$ $\text{Trans}(R) \mid \top \sqsubseteq \forall R^-.C_R \mid \top \sqsubseteq \forall R.C_R \mid a \in A \mid$ $\langle a, b \rangle \in R$

Layering on DLP

- A $\mathcal{DHL}$ ontology Φ and the corresponding logic program P_Φ agree on ground entailment (Herbrand)

Example

$$\Phi = \{ \quad (Male \sqcup Female) \sqcap \exists hasSpecies.\{human\} \sqsubseteq Person; \\ Person \sqsubseteq \forall hasName.String; \\ john \in Person; \langle john, \text{``}John''\rangle \in hasName \\ \}$$

$$P_\Phi = \{ \quad Person(x) \leftarrow Male(x), hasSpecies(x, human); \\ Person(x) \leftarrow Female(x), hasSpecies(x, human); \\ String(y) \leftarrow Person(x), hasName(x, y); \\ Person(john);\ hasName(john, \text{``}John'') \\ \}$$

Both Φ and P_Φ have as only ground entailments:
$Person(john);\ hasName(john, \text{``}John'');\ String(\text{``}John'')$

DLP and F-Logic Programs

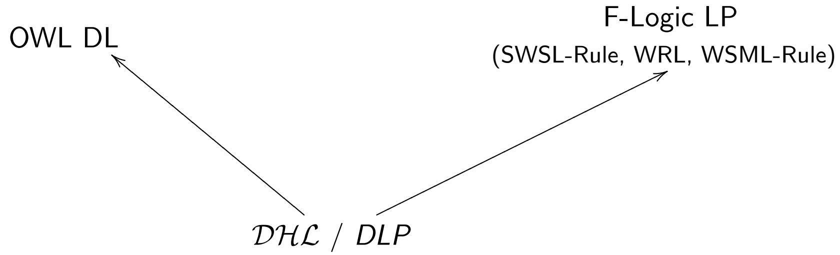
Example

$$\Phi = \{ \begin{array}{r} (Male \sqcup Female) \sqcap \exists hasSpecies.\{human\} \sqsubseteq Person; \\ Person \sqsubseteq \forall hasName.String; \\ john \in Person; \langle john, \text{``}John''\rangle \in hasName \end{array} \}$$

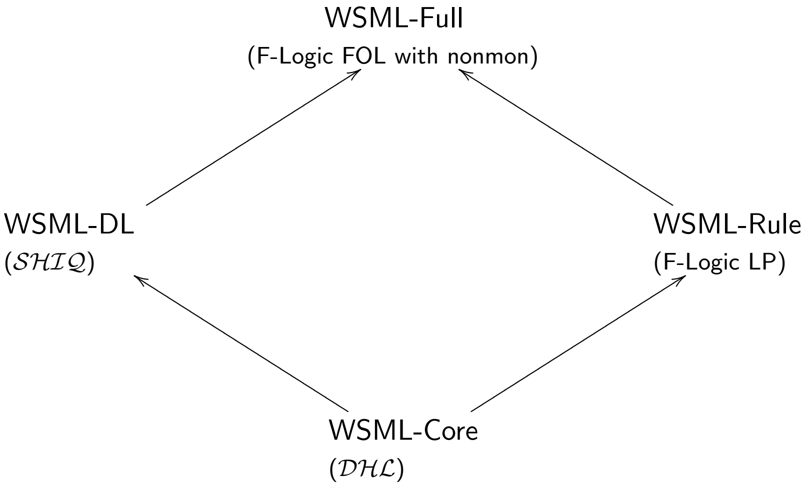
$$P_\Phi = \{ \begin{array}{r} x : Person \leftarrow x : Male, x[hasSpecies \twoheadrightarrow human]; \\ x : Person \leftarrow x : Female, x[hasSpecies \twoheadrightarrow human]; \\ y : String \leftarrow x : Person, x[hasName \twoheadrightarrow y]; \\ john : Person;\ john[hasName \twoheadrightarrow \text{``}John''] \end{array} \}$$

- P_Φ has as only ground entailments: $john : Person;\ john[hasName \twoheadrightarrow \text{``}John'']; \ \text{``}John'' : String$
- This corresponds to the ground entailments of Φ
- But, does this hold for all $\mathcal{DHL}$ ontologies?

The Semantic Web Languages Zoo



The Semantic Web Languages Zoo (con't.)



The Translation

Entity	**Predicate style**	**Frame style**
Class	$\delta(A(X))$	$X : A$
Property	$\delta(R(X,Y))$	$X[R \rightarrow\!\!\!\rightarrow Y]$
Equality	$\delta(X = Y)$	$X = Y$
n-ary predicate	$\delta(P(\vec{X}))$	$P(\vec{X})$
Universal	$\delta(\forall\vec{x}(\phi))$	$\forall\vec{x}(\delta(\phi))$
Existential	$\delta(\exists\vec{x}(\phi))$	$\exists\vec{x}(\delta(\phi))$
Conjunction	$\delta(\phi \wedge \psi)$	$(\delta(\phi) \wedge \delta(\psi))$
Disjunction	$\delta(\phi \vee \psi)$	$(\delta(\phi) \vee \delta(\psi))$
Implication	$\delta(\phi \supset \psi)$	$(\delta(\phi) \supset \delta(\psi))$
Negation	$\delta(\neg\phi)$	$\neg(\delta(\phi))$

Translation Example

$$\phi = (\forall x, y(x = y)) \supset (q(a) \leftrightarrow r(a)).$$

"If every individual is equal to every other, then the interpretations of q and r coincide."
ϕ is not a theorem of first-order logic.

$$\delta(\phi) = (\forall x, y(x = y)) \supset (a{:}q \leftrightarrow a{:}r).$$

"If every individual is equal to every other, then a is either a member of both q and r or of neither."
$\delta(\phi)$ is a theorem of F-Logic, because class identifiers are interpreted as individuals.

ϕ is not a cardinal formula.

Cardinal Formulas

Definition

$\phi \in \mathcal{L}$ is a formula and γ is the number of symbols in $\mathcal{L}$.
An interpretation $w = \langle U, \cdot^I \rangle$ is cardinal if $|U| \geq \gamma$.
ϕ is cardinal if the following holds:

> *If ϕ is true in every cardinal interpretation of $\mathcal{L}$, then ϕ is true in every interpretation of $\mathcal{L}$.*

Theorem

Let $\Phi \subseteq \mathcal{L}$ be a set of formulas and $\phi \in \mathcal{L}$ be a formula,

$$\textit{if} \quad \Phi \models \phi \quad \textit{then} \quad \delta(\Phi) \models_{\mathsf{f}} \delta(\phi).$$

If $\neg(\bigwedge \Phi) \vee \phi$ is cardinal, then also

$$\Phi \models \phi \quad \textit{iff} \quad \delta(\Phi) \models_{\mathsf{f}} \delta(\phi).$$

Cardinal Formulas (con't.)

- Definition of cardinal formulas is semantical
- Which classes of formulas are cardinal?

Lemma (Chen, Kifer, and Warren, 93)

The following classes of first-order formulas are cardinal.

1. *Sets of equality-free sentences, and*
2. *formulas of the form $\neg S$, where S is a conjunction of Horn clauses without equality in the head.*

Captures OWL DL without nominals, number restrictions, functional properties, and equality assertions.
Is sufficient for layering F-Logic on top of $\mathcal{DHL}$.
Can we do better? Yes!

$\mathcal{E}$-safe Formulas

Definition

$$I\mathcal{ESF} ::= \quad A \mid \neg A \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \forall \vec{x}(\chi \supset \phi) \mid \exists \vec{x}(\chi \wedge \phi)$$

A is an atom $p(\vec{t})$ or $t_1 = t_2$ with t_1, t_2 either both ground or non-ground terms;
ϕ, ϕ_1, ϕ_2 are $I\mathcal{E}$-safe formulas;
χ is an atom $p(\vec{t})$ or a conjunction of atoms of the form $p(\vec{t})$ such that the variable graph of χ is connected;
every free variable in ϕ must appear in χ.

$$\mathcal{ESF} ::= \quad \varphi \mid \forall x(\phi) \mid \exists x(\phi) \mid \psi_1 \wedge \psi_2 \mid \psi_1 \vee \psi_2$$

ψ_1, ψ_2 are $\mathcal{E}$-safe formulas;
ϕ, φ are $I\mathcal{E}$-safe formulas;
x is the only free variable in ϕ.

$\mathcal{E}$-safe Formulas (con't.)

Example

The following formulas are $\mathcal{E}$-safe:

$\forall x(p(x) \supset q(x))$

$\forall x(s(x, y) \supset p(x))$

$\exists x, y(p(x) \wedge r(x, y) \wedge x = y)$

$\forall x(r(x))$

The following formulas are not $\mathcal{E}$-safe:

$\forall x, y(x = y)$

$\forall x, y(a(x) \wedge a(y) \supset x = y)$

$\forall x, y(x = y \supset p(x, y))$

$\forall x(x = a)$

$\forall x(x = a)$ is equivalent to the $\mathcal{SHOIQ}$ axiom $\top \sqsubseteq \{a\}$, thus $\mathcal{SHOIQ}$ is not $\mathcal{E}$-safe.

$\mathcal{E}$-safe formulas are cardinal

Lemma

The class of $\mathcal{E}$-safe sentences is cardinal.

$\mathcal{SHIQ}$ formulas are $\mathcal{E}$-safe

Theorem

Any (negation of a) $\mathcal{SHIQ}$ axiom ϕ can be rewritten to an $\mathcal{E}$-safe formula ϕ' such that ϕ and ϕ' are equivalent, i.e., share the same models.

Corollary

Let Φ be a set of $\mathcal{SHIQ}$ axioms and ϕ a $\mathcal{SHIQ}$ axiom, then

$$\Phi \models \phi \quad \textit{iff} \quad \delta(\Phi) \models_{\mathsf{f}} \delta(\phi).$$

Establishes layering of WSML-Full on top of WSML-DL.

Questions?