Translating Ontologies from Predicate-based to Frame-based Languages

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RuleML 2006

2006-11-10
The Semantic Web Languages Zoo

Translating Predicate-based Ontologies to F-Logic

The Translation
Cardinal Formulas
Equality-safe Formulas
$SHIQ$ Layering
Description Logic Programs (DLP)

- Intersection of $SHOIN$ (OWL DL) and Logic Programming
- Essentially, the Horn subset of $SHOIN$: $DHL$ (Description Horn Logic)

$\mathcal{DHL}$ descriptions:

$C, D \quad \rightarrow \quad A \mid C \cap D \mid \exists R.\{o\}$

$C_L, D_L \quad \rightarrow \quad C \mid C_L \sqcup D_L \mid \exists R.C_L \mid \geq 1R_L \mid \{o_1, \ldots, o_n\}$

$C_R, D_R \quad \rightarrow \quad C \mid \forall R.C_R$

$\mathcal{DHL}$ axioms:

$C_L \sqsubseteq D_R \mid C \equiv D \mid R \sqsubseteq S \mid R \equiv S \mid R \equiv S^- \mid \text{Trans}(R) \mid \top \sqsubseteq \forall R^- . C_R \mid \top \sqsubseteq \forall R.C_R \mid a \in A \mid \langle a, b \rangle \in R$
Layering on DLP

- A $\mathcal{DH\!L}$ ontology $\Phi$ and the corresponding logic program $P_\Phi$ agree on ground entailment (Herbrand)

Example

$\Phi = \{ (\text{Male} \sqcup \text{Female}) \sqcap \exists \text{hasSpecies}.\{\text{human}\} \subseteq \text{Person}; \\
\text{Person} \subseteq \forall \text{hasName}.\text{String}; \\
\text{john} \in \text{Person}; \langle \text{john}, \text{"John"} \rangle \in \text{hasName} \}
$

$P_\Phi = \{ \text{Person}(x) \leftarrow \text{Male}(x), \text{hasSpecies}(x, \text{human}); \\
\text{Person}(x) \leftarrow \text{Female}(x), \text{hasSpecies}(x, \text{human}); \\
\text{String}(y) \leftarrow \text{Person}(x), \text{hasName}(x, y); \\
\text{Person}(\text{john}); \text{hasName}(\text{john}, \text{"John"}) \}
$

Both $\Phi$ and $P_\Phi$ have as only ground entailments:

$\text{Person}(\text{john}); \text{hasName}(\text{john}, \text{"John"}); \text{String}(\text{"John"})$
DLP and F-Logic Programs

Example

\[ \Phi = \{ (\text{Male} \sqcup \text{Female}) \sqcap \exists \text{hasSpecies.}\{\text{human}\} \sqsubseteq \text{Person}; \]
\[ \text{Person} \sqsubseteq \forall \text{hasName}.\text{String}; \]
\[ \text{john} \in \text{Person}; \langle \text{john}, \text{"John"} \rangle \in \text{hasName} \}
\]

\[ P_\Phi = \{ x : \text{Person} \leftarrow x : \text{Male}, x[\text{hasSpecies} \rightarrow \text{human}]; \]
\[ x : \text{Person} \leftarrow x : \text{Female}, x[\text{hasSpecies} \rightarrow \text{human}]; \]
\[ y : \text{String} \leftarrow x : \text{Person}, x[\text{hasName} \rightarrow y]; \]
\[ \text{john} : \text{Person}; \text{john}[\text{hasName} \rightarrow \text{"John"}] \}
\]

\[ P_\Phi \text{ has as only ground entailments:} \]
\[ \text{john} : \text{Person}; \text{john}[\text{hasName} \rightarrow \text{"John"}]; \text{"John"} : \text{String} \]
\[ \text{This corresponds to the ground entailments of } \Phi \]
\[ \text{But, does this hold for all } DHL \text{ ontologies?} \]
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OWL DL

\( \mathcal{DHL} \) / \( \mathcal{DLP} \)

F-Logic LP

(SWSL-Rule, WRL, WSML-Rule)
The Semantic Web Languages Zoo (con’t.)

- **WSML-Full**
  - (F-Logic FOL with nonmon)

- **WSML-DL**
  - (*SHIQ*)

- **WSML-Core**
  - (*DHL*)

- **WSML-Rule**
  - (F-Logic LP)
## The Translation

<table>
<thead>
<tr>
<th>Entity</th>
<th>Predicate style</th>
<th>Frame style</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>(\delta(A(X)))</td>
<td>(X : A)</td>
</tr>
<tr>
<td>Property</td>
<td>(\delta(R(X, Y)))</td>
<td>(X[R\rightarrow Y])</td>
</tr>
<tr>
<td>Equality</td>
<td>(\delta(X = Y))</td>
<td>(X = Y)</td>
</tr>
<tr>
<td>(n)-ary predicate</td>
<td>(\delta(P(\vec{X})))</td>
<td>(P(\vec{X}))</td>
</tr>
<tr>
<td>Universal</td>
<td>(\delta(\forall \vec{x}(\phi)))</td>
<td>(\forall \vec{x}(\delta(\phi)))</td>
</tr>
<tr>
<td>Existential</td>
<td>(\delta(\exists \vec{x}(\phi)))</td>
<td>(\exists \vec{x}(\delta(\phi)))</td>
</tr>
<tr>
<td>Conjunction</td>
<td>(\delta(\phi \land \psi))</td>
<td>(\delta(\phi) \land \delta(\psi))</td>
</tr>
<tr>
<td>Disjunction</td>
<td>(\delta(\phi \lor \psi))</td>
<td>(\delta(\phi) \lor \delta(\psi))</td>
</tr>
<tr>
<td>Implication</td>
<td>(\delta(\phi \supset \psi))</td>
<td>(\delta(\phi) \supset \delta(\psi))</td>
</tr>
<tr>
<td>Negation</td>
<td>(\delta(\neg \phi))</td>
<td>(\neg(\delta(\phi)))</td>
</tr>
</tbody>
</table>
Translation Example

\[ \phi = (\forall x, y (x = y)) \supset (q(a) \leftrightarrow r(a)). \]

“If every individual is equal to every other, then the interpretations of \( q \) and \( r \) coincide.”

\( \phi \) is not a theorem of first-order logic.

\[ \delta(\phi) = (\forall x, y (x = y)) \supset (a:q \leftrightarrow a:r). \]

“If every individual is equal to every other, then \( a \) is either a member of both \( q \) and \( r \) or of neither.”

\( \delta(\phi) \) is a theorem of F-Logic, because class identifiers are interpreted as individuals.

\( \phi \) is not a cardinal formula.
Cardinal Formulas

Definition
\( \phi \in \mathcal{L} \) is a formula and \( \gamma \) is the number of symbols in \( \mathcal{L} \).
An interpretation \( w = \langle U, \cdot^I \rangle \) is cardinal if \( |U| \geq \gamma \).
\( \phi \) is cardinal if the following holds:

\[
\text{If } \phi \text{ is true in every cardinal interpretation of } \mathcal{L}, \text{ then } \phi \text{ is true in every interpretation of } \mathcal{L}.
\]

Theorem
Let \( \Phi \subseteq \mathcal{L} \) be a set of formulas and \( \phi \in \mathcal{L} \) be a formula,

\[
\text{if } \Phi \models \phi \text{ then } \delta(\Phi) \models_f \delta(\phi).
\]

If \( \neg(\bigwedge \Phi) \lor \phi \) is cardinal, then also

\[
\Phi \models \phi \text{ iff } \delta(\Phi) \models_f \delta(\phi).
\]
Definition of cardinal formulas is semantical
Which classes of formulas are cardinal?

Lemma (Chen, Kifer, and Warren, 93)

The following classes of first-order formulas are cardinal.

1. Sets of equality-free sentences, and
2. formulas of the form $\neg S$, where $S$ is a conjunction of Horn clauses without equality in the head.

Captures OWL DL without nominals, number restrictions, functional properties, and equality assertions.
Is sufficient for layering F-Logic on top of DHL.

Can we do better? Yes!
\( \mathcal{E}\mbox{-safe} \) Formulas

**Definition**

\[
\mathcal{I}\mathcal{E}\mathcal{S}\mathcal{F} ::= \ A \mid \lnot A \mid \phi_1 \land \phi_2 \mid \phi_1 \lor \phi_2 \mid \forall \vec{x} (\chi \supset \phi) \mid \exists \vec{x} (\chi \land \phi)
\]

\( A \) is an atom \( p(\vec{t}) \) or \( t_1 = t_2 \) with \( t_1, t_2 \) either both ground or non-ground terms;
\( \phi, \phi_1, \phi_2 \) are \( \mathcal{I}\mathcal{E} \)-safe formulas;
\( \chi \) is an atom \( p(\vec{t}) \) or a conjunction of atoms of the form \( p(\vec{t}) \) such that the variable graph of \( \chi \) is connected;
every free variable in \( \phi \) must appear in \( \chi \).

\[
\mathcal{E}\mathcal{S}\mathcal{F} ::= \ \varphi \mid \forall x (\phi) \mid \exists x (\phi) \mid \psi_1 \land \psi_2 \mid \psi_1 \lor \psi_2
\]

\( \psi_1, \psi_2 \) are \( \mathcal{E} \)-safe formulas;
\( \phi, \varphi \) are \( \mathcal{I}\mathcal{E} \)-safe formulas;
\( x \) is the only free variable in \( \phi \).
Example

The following formulas are $\mathcal{E}$-safe:
\[
\forall x (p(x) \supset q(x)) \\
\forall x (s(x, y) \supset p(x)) \\
\exists x, y (p(x) \land r(x, y) \land x = y) \\
\forall x (r(x))
\]

The following formulas are not $\mathcal{E}$-safe:
\[
\forall x, y (x = y) \\
\forall x, y (a(x) \land a(y) \supset x = y) \\
\forall x, y (x = y \supset p(x, y)) \\
\forall x (x = a)
\]

$\forall x (x = a)$ is equivalent to the $\text{SHOIQ}$ axiom $\top \sqsubseteq \{a\}$, thus $\text{SHOIQ}$ is not $\mathcal{E}$-safe.
Lemma

The class of $\mathcal{E}$-safe sentences is cardinal.
**SHIQ** formulas are $\mathcal{E}$-safe

**Theorem**
Any (negation of a) **SHIQ** axiom $\phi$ can be rewritten to an $\mathcal{E}$-safe formula $\phi'$ such that $\phi$ and $\phi'$ are equivalent, i.e., share the same models.

**Corollary**
Let $\Phi$ be a set of **SHIQ** axioms and $\phi$ a **SHIQ** axiom, then

$$\Phi \models \phi \iff \delta(\Phi) \models_{f} \delta(\phi).$$

Establishes layering of WSML-Full on top of WSML-DL.
Questions?