

Translating Ontologies from Predicate-based to Frame-based Languages

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Outline

The Semantic Web Languages Zoo

Translating Predicate-based Ontologies to F-Logic

- The Translation

- Cardinal Formulas

- Equality-safe Formulas

- SHIQ* Layering

Description Logic Programs (DLP)

- ▶ Intersection of *SHOIN* (OWL DL) and Logic Programming
- ▶ Essentially, the Horn subset of *SHOIN*: *DHL* (Description Horn Logic)

- ▶ *DHL* descriptions:

$$C, D \longrightarrow A \mid C \sqcap D \mid \exists R. \{o\}$$

$$C_L, D_L \longrightarrow C \mid C_L \sqcup D_L \mid \exists R. C_L \mid \geq 1 R_L \mid \{o_1, \dots, o_n\}$$

$$C_R, D_R \longrightarrow C \mid \forall R. C_R$$

- ▶ *DHL* axioms:

$$C_L \sqsubseteq D_R \mid C \equiv D \mid R \sqsubseteq S \mid R \equiv S \mid R \equiv S^- \mid \text{Trans}(R) \mid \top \sqsubseteq \forall R^-. C_R \mid \top \sqsubseteq \forall R. C_R \mid a \in A \mid \langle a, b \rangle \in R$$

Layering on DLP

- ▶ A \mathcal{DHL} ontology Φ and the corresponding logic program P_Φ agree on ground entailment (Herbrand)

Example

$$\Phi = \left\{ \begin{array}{l} (Male \sqcup Female) \sqcap \exists hasSpecies.\{human\} \sqsubseteq Person; \\ Person \sqsubseteq \forall hasName.String; \\ john \in Person; \langle john, "John" \rangle \in hasName \end{array} \right\}$$
$$P_\Phi = \left\{ \begin{array}{l} Person(x) \leftarrow Male(x), hasSpecies(x, human); \\ Person(x) \leftarrow Female(x), hasSpecies(x, human); \\ String(y) \leftarrow Person(x), hasName(x, y); \\ Person(john); hasName(john, "John") \end{array} \right\}$$

Both Φ and P_Φ have as only ground entailments:

$Person(john); hasName(john, "John"); String("John")$

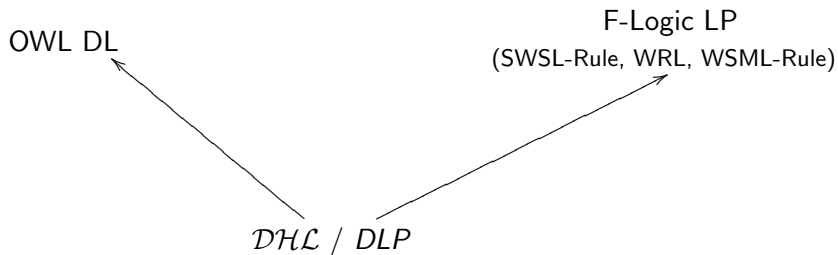
DLP and F-Logic Programs

Example

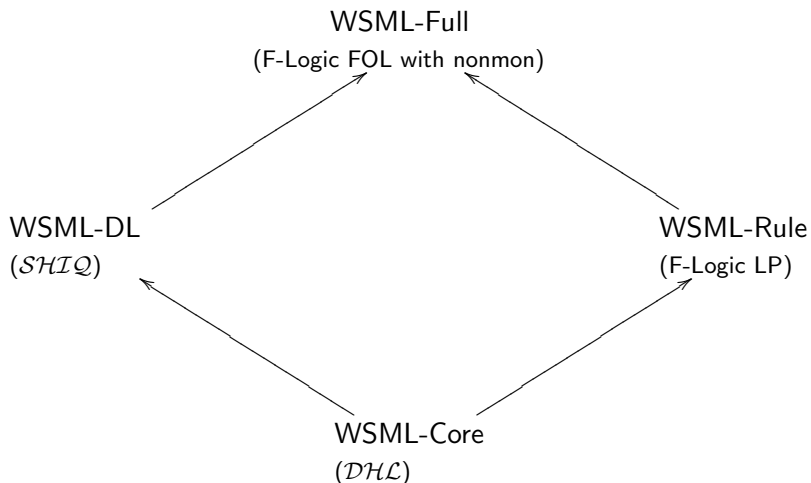
$$\Phi = \left\{ \begin{array}{l} (Male \sqcup Female) \sqcap \exists hasSpecies.\{human\} \sqsubseteq Person; \\ \phantom{(Male \sqcup Female) \sqcap \exists hasSpecies.\{human\} \sqsubseteq Person;} Person \sqsubseteq \forall hasName.String; \\ \phantom{(Male \sqcup Female) \sqcap \exists hasSpecies.\{human\} \sqsubseteq Person;} john \in Person; \langle john, "John" \rangle \in hasName \end{array} \right\}$$
$$P_{\Phi} = \left\{ \begin{array}{l} x : Person \leftarrow x : Male, x[hasSpecies \twoheadrightarrow human]; \\ x : Person \leftarrow x : Female, x[hasSpecies \twoheadrightarrow human]; \\ y : String \leftarrow x : Person, x[hasName \twoheadrightarrow y]; \\ john : Person; john[hasName \twoheadrightarrow "John"] \end{array} \right\}$$

- ▶ P_{Φ} has as only ground entailments:
 $john : Person; john[hasName \twoheadrightarrow "John"]; "John" : String$
- ▶ This corresponds to the ground entailments of Φ
- ▶ **But**, does this hold for all DHL ontologies?

The Semantic Web Languages Zoo



The Semantic Web Languages Zoo (con't.)



The Translation

Entity	Predicate style	Frame style
Class	$\delta(A(X))$	$X : A$
Property	$\delta(R(X, Y))$	$X[R \rightarrow Y]$
Equality	$\delta(X = Y)$	$X = Y$
n -ary predicate	$\delta(P(\vec{X}))$	$P(\vec{X})$
Universal	$\delta(\forall \vec{x}(\phi))$	$\forall \vec{x}(\delta(\phi))$
Existential	$\delta(\exists \vec{x}(\phi))$	$\exists \vec{x}(\delta(\phi))$
Conjunction	$\delta(\phi \wedge \psi)$	$(\delta(\phi) \wedge \delta(\psi))$
Disjunction	$\delta(\phi \vee \psi)$	$(\delta(\phi) \vee \delta(\psi))$
Implication	$\delta(\phi \supset \psi)$	$(\delta(\phi) \supset \delta(\psi))$
Negation	$\delta(\neg \phi)$	$\neg(\delta(\phi))$

Translation Example

$$\phi = (\forall x, y(x = y)) \supset (q(a) \leftrightarrow r(a)).$$

“If every individual is equal to every other, then the interpretations of q and r coincide.”

ϕ is not a theorem of first-order logic.

$$\delta(\phi) = (\forall x, y(x = y)) \supset (a: q \leftrightarrow a: r).$$

“If every individual is equal to every other, then a is either a member of both q and r or of neither.”

$\delta(\phi)$ is a theorem of F-Logic, because class identifiers are interpreted as individuals.

ϕ is not a cardinal formula.

Cardinal Formulas

Definition

$\phi \in \mathcal{L}$ is a formula and γ is the number of symbols in \mathcal{L} .

An interpretation $w = \langle U, \cdot^I \rangle$ is cardinal if $|U| \geq \gamma$.

ϕ is cardinal if the following holds:

If ϕ is true in every cardinal interpretation of \mathcal{L} , then ϕ is true in every interpretation of \mathcal{L} .

Theorem

Let $\Phi \subseteq \mathcal{L}$ be a set of formulas and $\phi \in \mathcal{L}$ be a formula,

if $\Phi \models \phi$ then $\delta(\Phi) \models_f \delta(\phi)$.

If $\neg(\bigwedge \Phi) \vee \phi$ is cardinal, then also

$\Phi \models \phi$ iff $\delta(\Phi) \models_f \delta(\phi)$.

Cardinal Formulas (con't.)

- ▶ Definition of cardinal formulas is semantical
- ▶ Which classes of formulas are cardinal?

Lemma (Chen, Kifer, and Warren, 93)

The following classes of first-order formulas are cardinal.

1. *Sets of equality-free sentences, and*
2. *formulas of the form $\neg S$, where S is a conjunction of Horn clauses without equality in the head.*

Captures OWL DL without nominals, number restrictions, functional properties, and equality assertions.

Is sufficient for layering F-Logic on top of DHL .

Can we do better? **Yes!**

\mathcal{E} -safe Formulas

Definition

$$l\mathcal{E}\mathcal{S}\mathcal{F} ::= A \mid \neg A \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \\ \forall \vec{x}(\chi \supset \phi) \mid \exists \vec{x}(\chi \wedge \phi)$$

A is an atom $p(\vec{t})$ or $t_1 = t_2$ with t_1, t_2 either both ground or non-ground terms;

ϕ, ϕ_1, ϕ_2 are $l\mathcal{E}$ -safe formulas;

χ is an atom $p(\vec{t})$ or a conjunction of atoms of the form $p(\vec{t})$ such that the variable graph of χ is connected;

every free variable in ϕ must appear in χ .

$$\mathcal{E}\mathcal{S}\mathcal{F} ::= \varphi \mid \forall x(\phi) \mid \exists x(\phi) \mid \psi_1 \wedge \psi_2 \mid \psi_1 \vee \psi_2$$

ψ_1, ψ_2 are \mathcal{E} -safe formulas;

ϕ, φ are $l\mathcal{E}$ -safe formulas;

x is the only free variable in ϕ .

\mathcal{E} -safe Formulas (con't.)

Example

The following formulas are \mathcal{E} -safe:

$$\forall x(p(x) \supset q(x))$$

$$\forall x(s(x, y) \supset p(x))$$

$$\exists x, y(p(x) \wedge r(x, y) \wedge x = y)$$

$$\forall x(r(x))$$

The following formulas are not \mathcal{E} -safe:

$$\forall x, y(x = y)$$

$$\forall x, y(a(x) \wedge a(y) \supset x = y)$$

$$\forall x, y(x = y \supset p(x, y))$$

$$\forall x(x = a)$$

$\forall x(x = a)$ is equivalent to the *SHOIQ* axiom $\top \sqsubseteq \{a\}$, thus *SHOIQ* is not \mathcal{E} -safe.

\mathcal{E} -safe formulas are cardinal

Lemma

The class of \mathcal{E} -safe sentences is cardinal.

SHIQ formulas are \mathcal{E} -safe

Theorem

Any (negation of a) $SHIQ$ axiom ϕ can be rewritten to an \mathcal{E} -safe formula ϕ' such that ϕ and ϕ' are equivalent, i.e., share the same models.

Corollary

Let Φ be a set of $SHIQ$ axioms and ϕ a $SHIQ$ axiom, then

$$\Phi \models \phi \quad \text{iff} \quad \delta(\Phi) \models_f \delta(\phi).$$

Establishes layering of WSML-Full on top of WSML-DL.

Questions?